Dynamic disease models

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What is a model?

A mathematical model is an simplified mathematical description of a more complex system.

which means....

All models are wrong, but some are useful

- George Box

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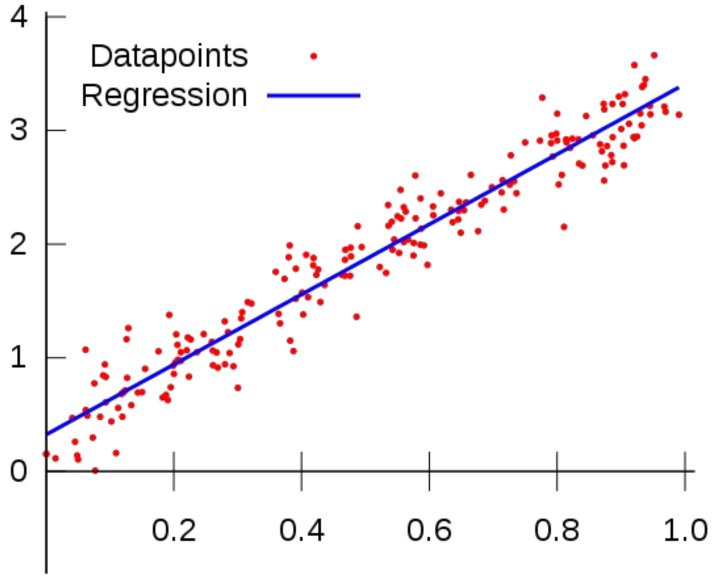
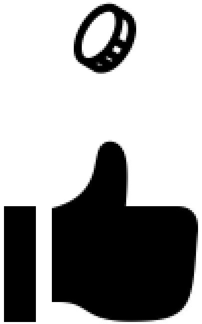
$$y_i = \beta_0 + \beta_1 x_{1,i} + \ldots + \beta_n x_{n,i}$$



Why do we need different tools?

Why not regression?

Independence



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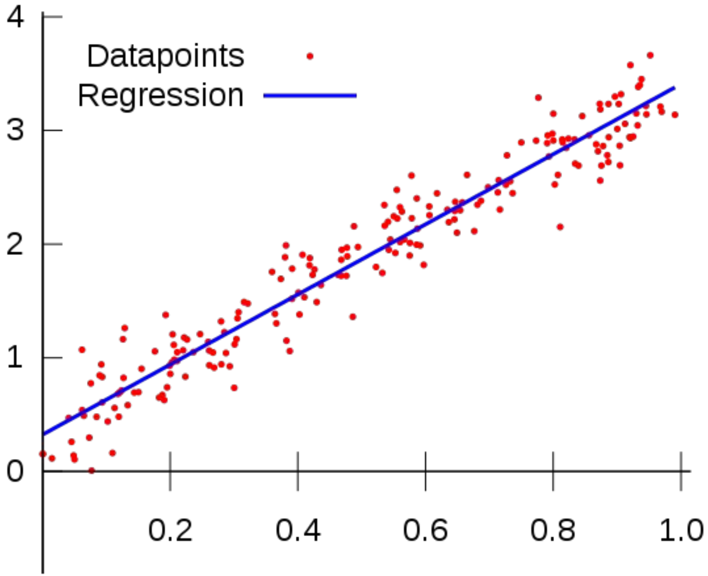


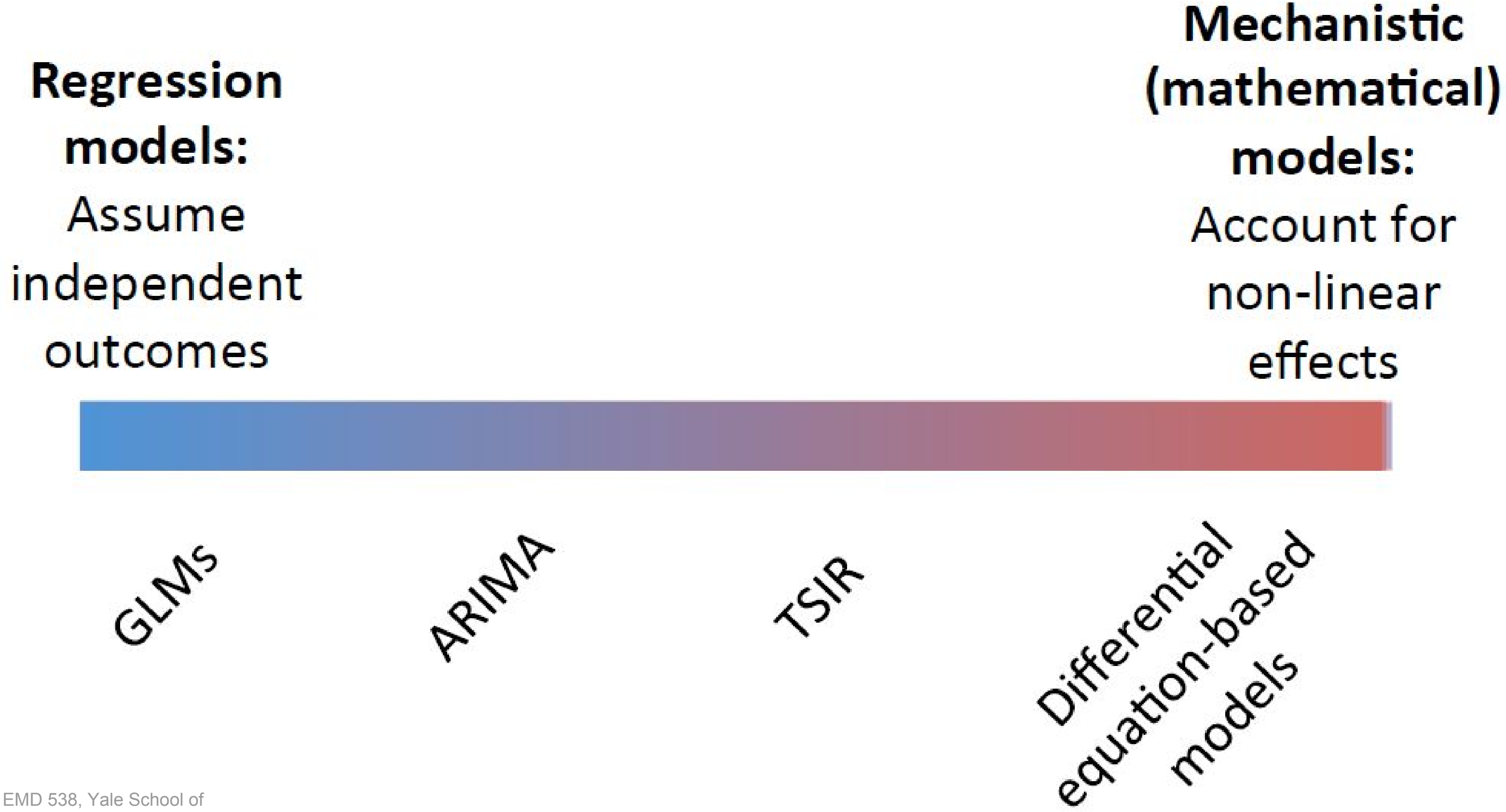
How infectious diseases are different
(and why we need dynamic models)

1. Treating an individual can affect others (non-independence)

2. Conflicts between individual and group interests

Adapted from Virginia Pitzer’s lecture on Infectious disease modeling: Yale School of Public Health 2014

Model spectrum



From: Pitzer, EMD 538, Yale School of Public Health

Model spectrum

Regression models:
Assume independent outcomes

Mechanistic (mathematical) models:
Account for non-linear effects

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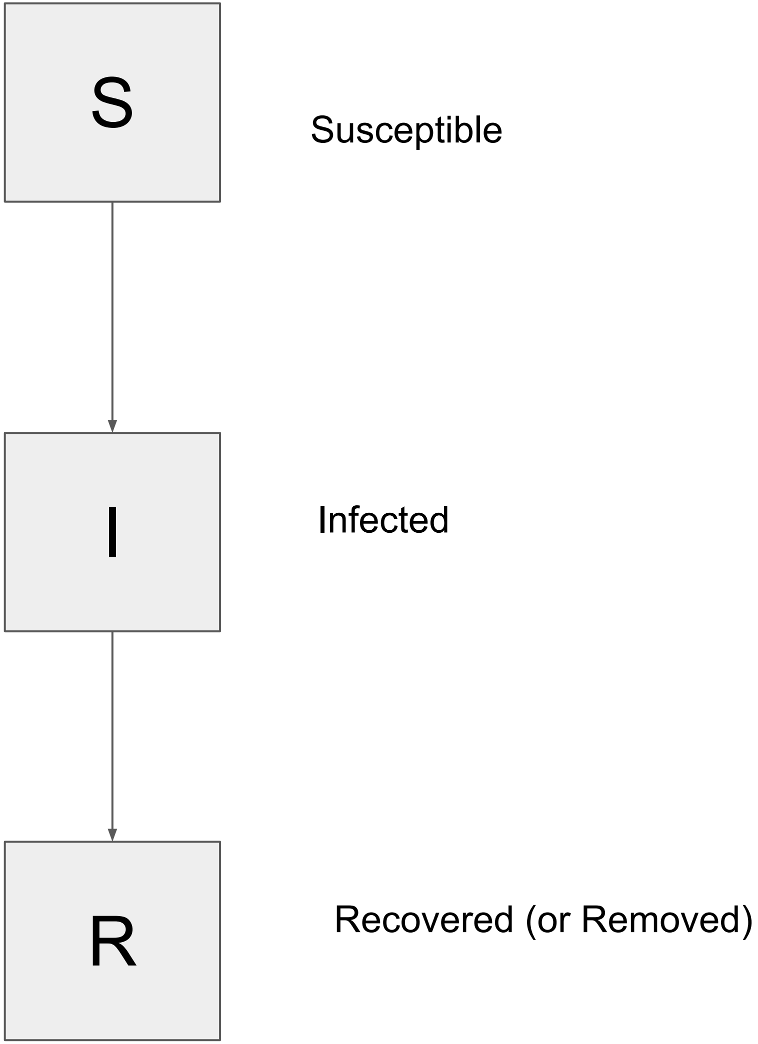
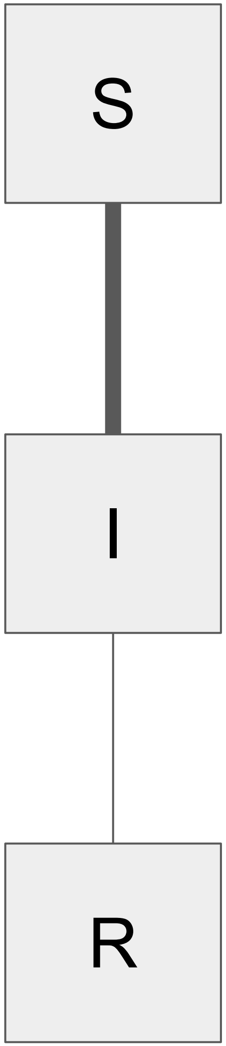
S-I-R model Basics



Big pipes = easier to move



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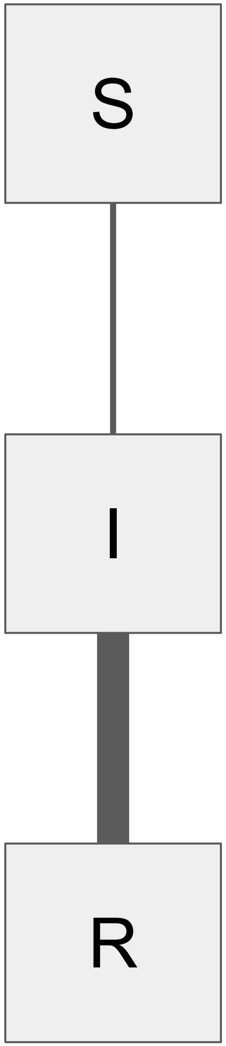
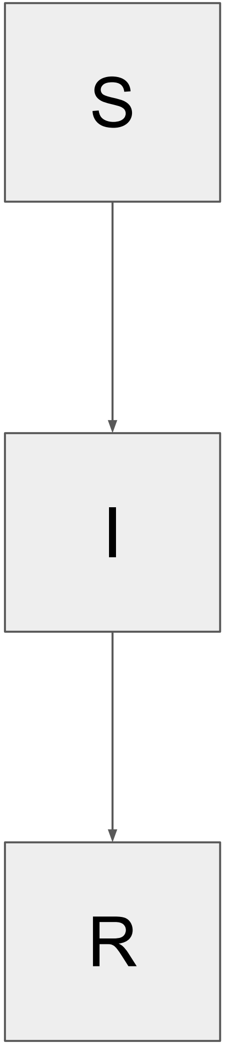


Describe flow rate mathematically



contact rate * prob(transmission per contact) * number of ppl susceptible * prob of being infectious

1 per minute * 0.5 per contact * 95 people * 5 people / 100 people =

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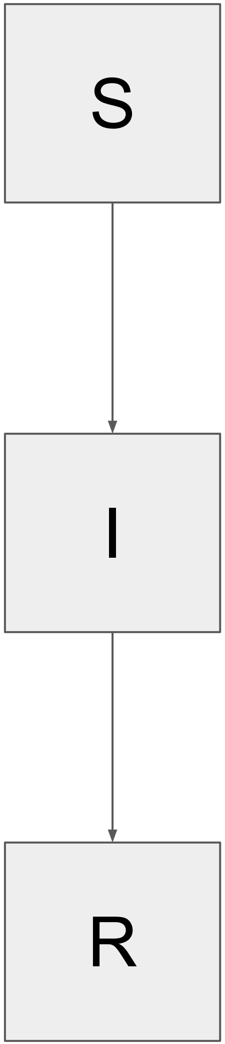


contact rate * prob(transmission per contact) * number of ppl susceptible * prob of being infectious

1 per minute * 0.5 per contact * 95 people * 5 people / 100 people = 2.4

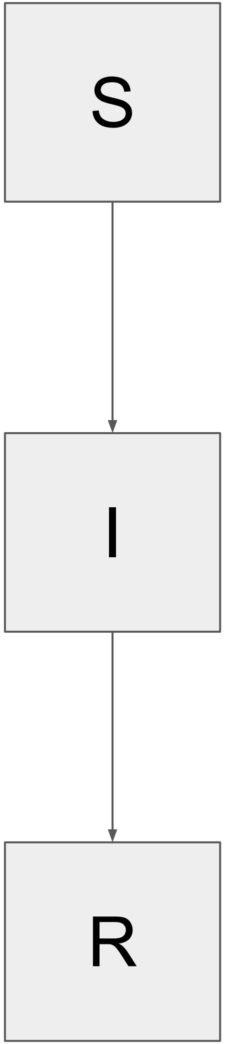
1 per minute * 0.5 per contact * 95(-2) people * 5 (+2) people / 100 people = 3.3

β = “effective contact rate”

= (contact rate) x (prob. of transmission per contact)

γ = “recovery rate”

= 1 / (duration of infection)



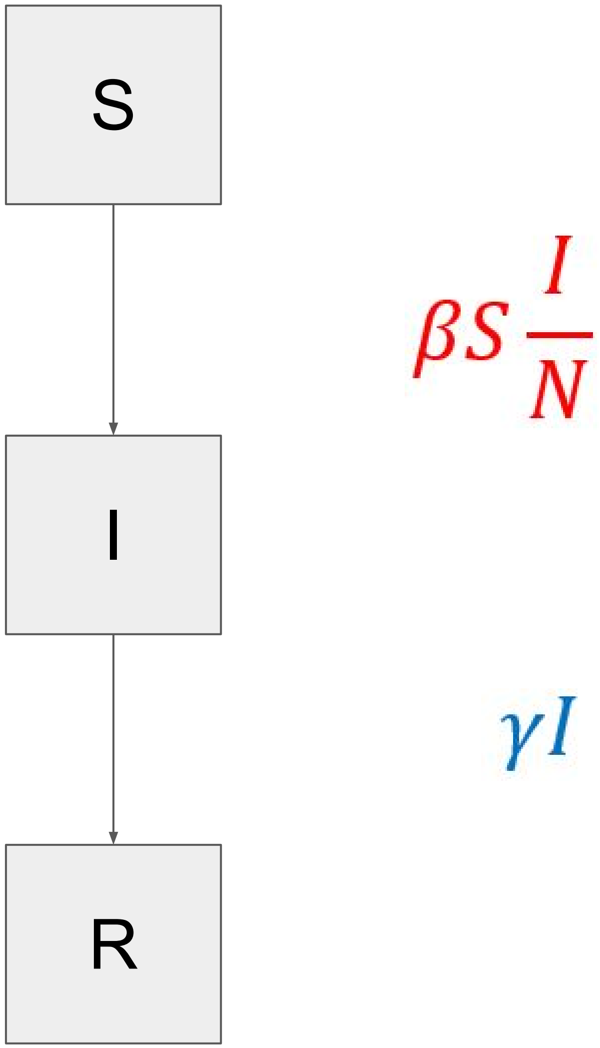
$$\beta S \frac{I}{N}$$

$$\gamma I$$

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$



SIR Model assumptions

1. Homogenous mixing of people
2. Large population
3. Closed population (no births, no deaths)
4. Acquired immunity is lifelong
5. People are infectious as soon as they are infected

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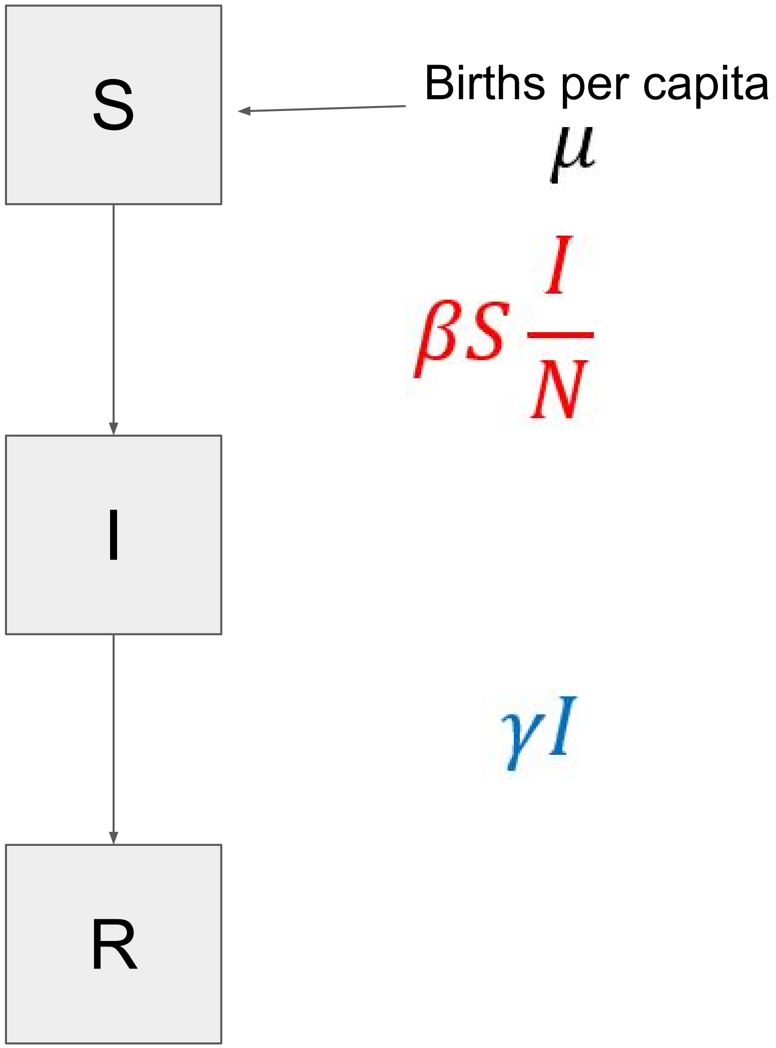
There are ways to get around these

Expanding the SIR model

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I$$

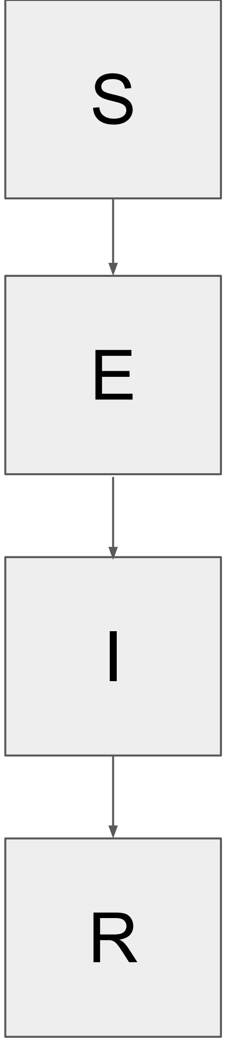
$$\frac{dR}{dt} = \gamma I$$



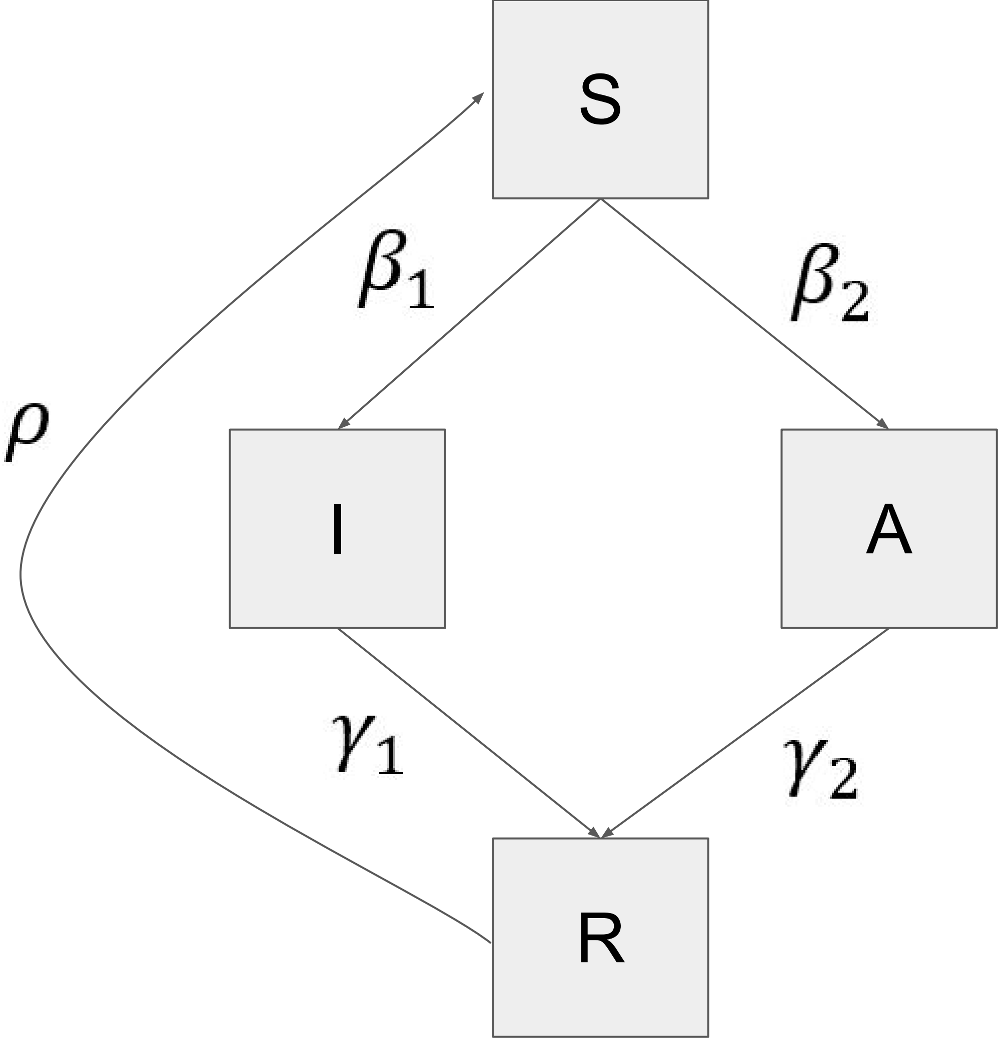
Asymptomatic infections

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$

$$\frac{dE}{dt} = \beta S \frac{I}{N} - \theta E$$

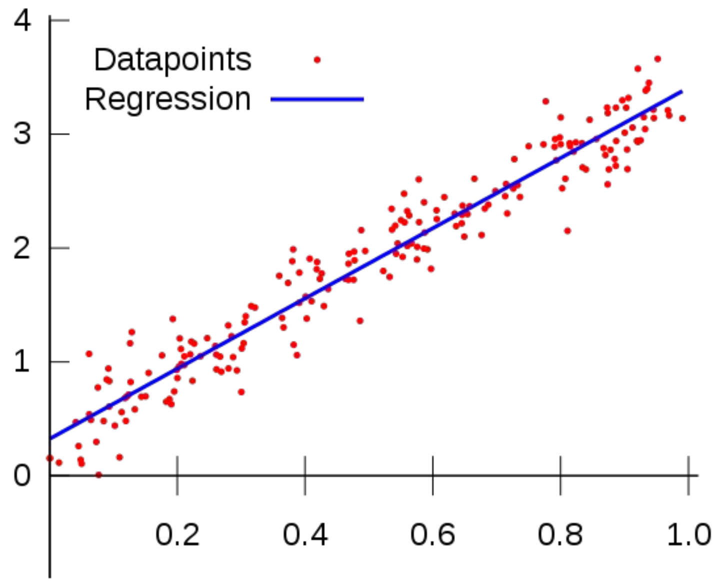


Waning immunity



Lab part 1

Mechanistic vs Stochastic



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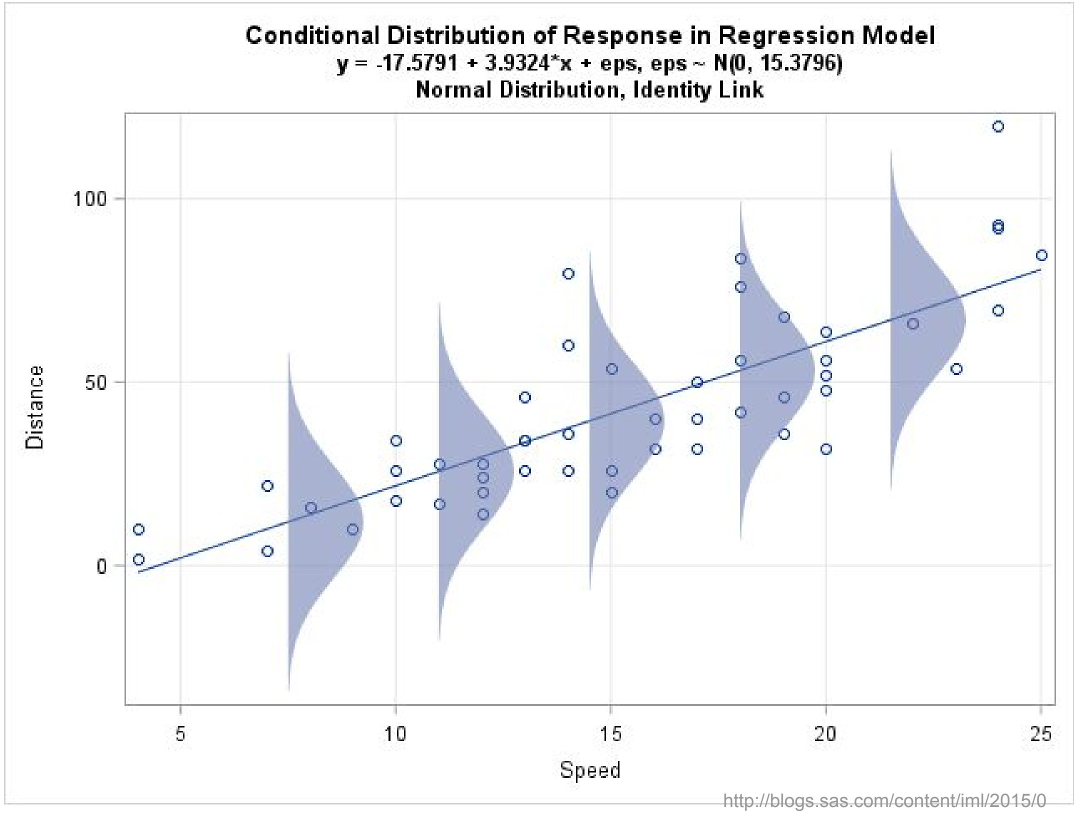


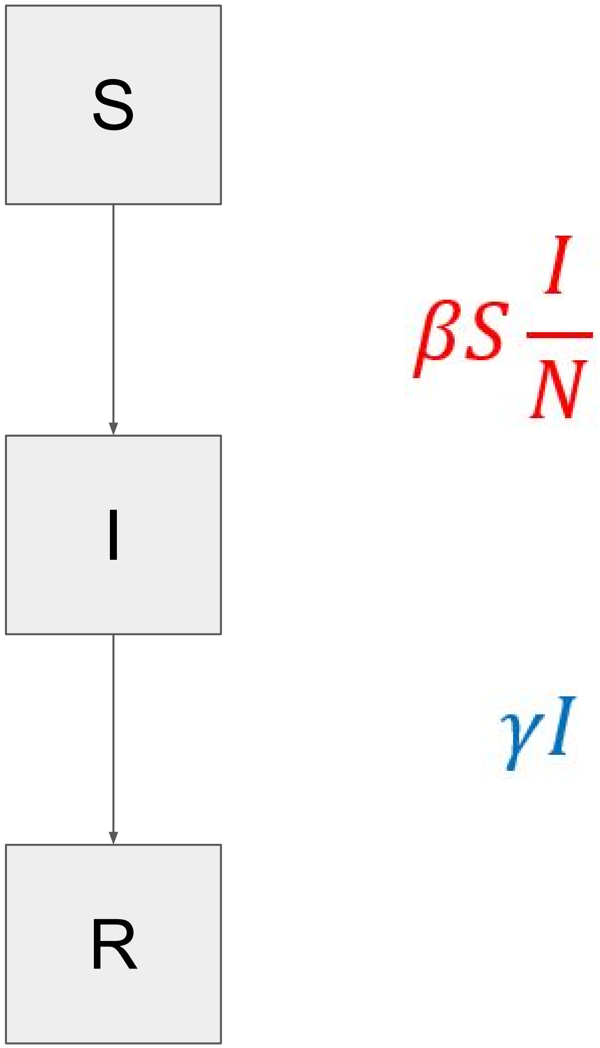
http://blogs.sas.com/content/iml/2015/09/16/plot-distrib-exp.html#prettyPhoto

Mechanistic vs Stochastic

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

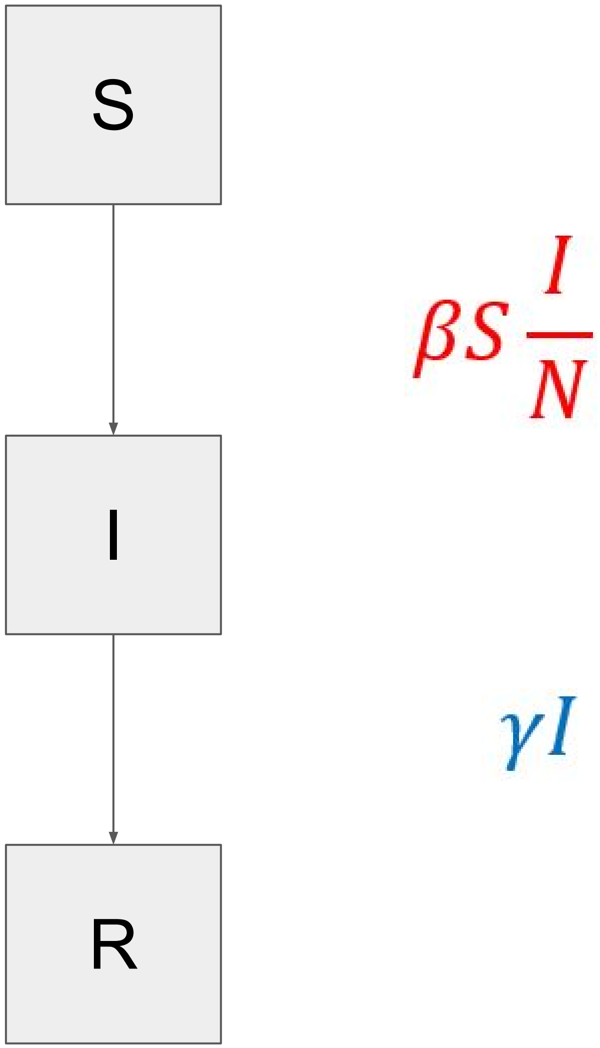


Mechanistic vs Stochastic

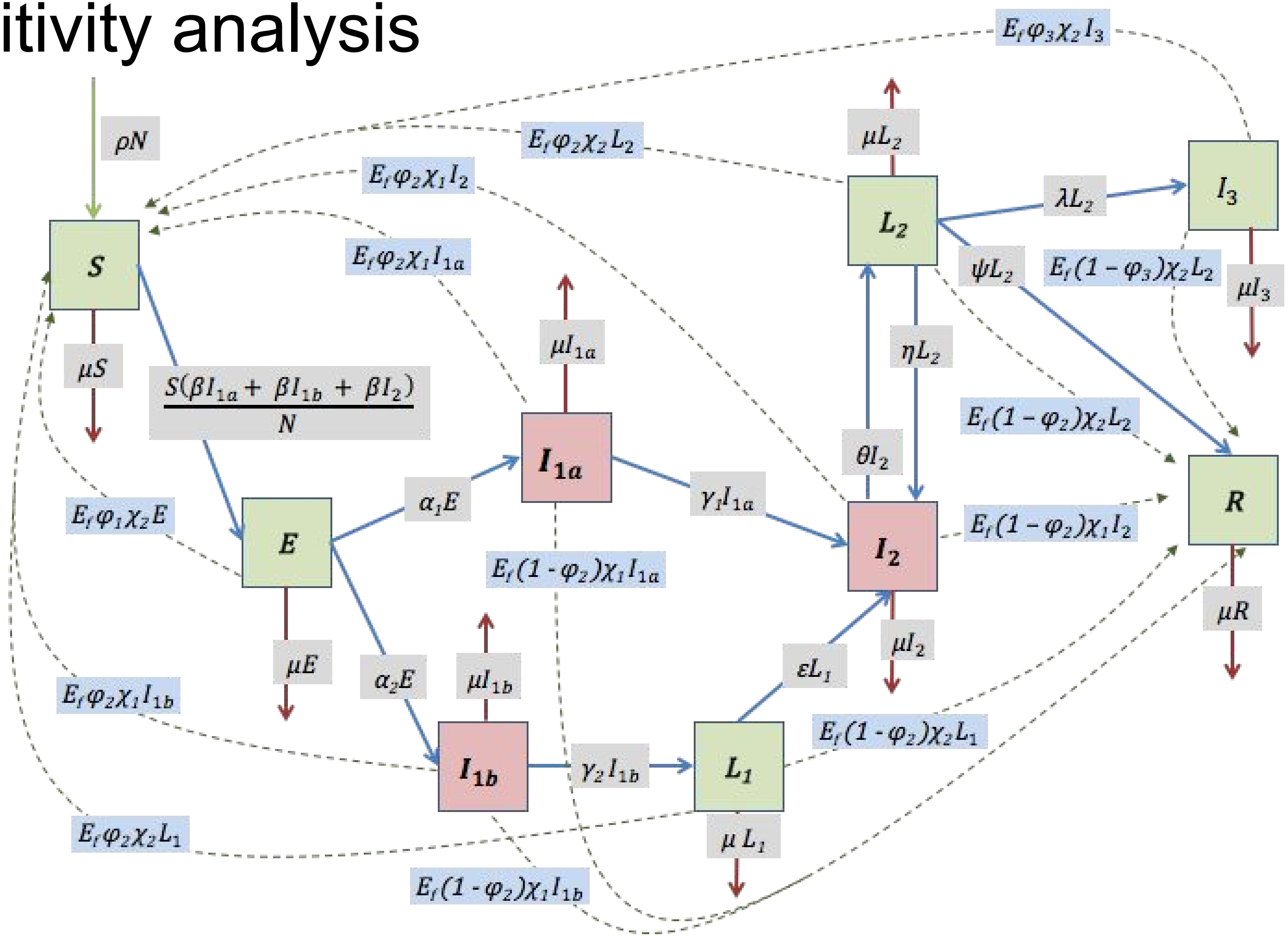
$$S_{t+1} = S_t - rpois(\beta S \frac{I}{N})$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I$$

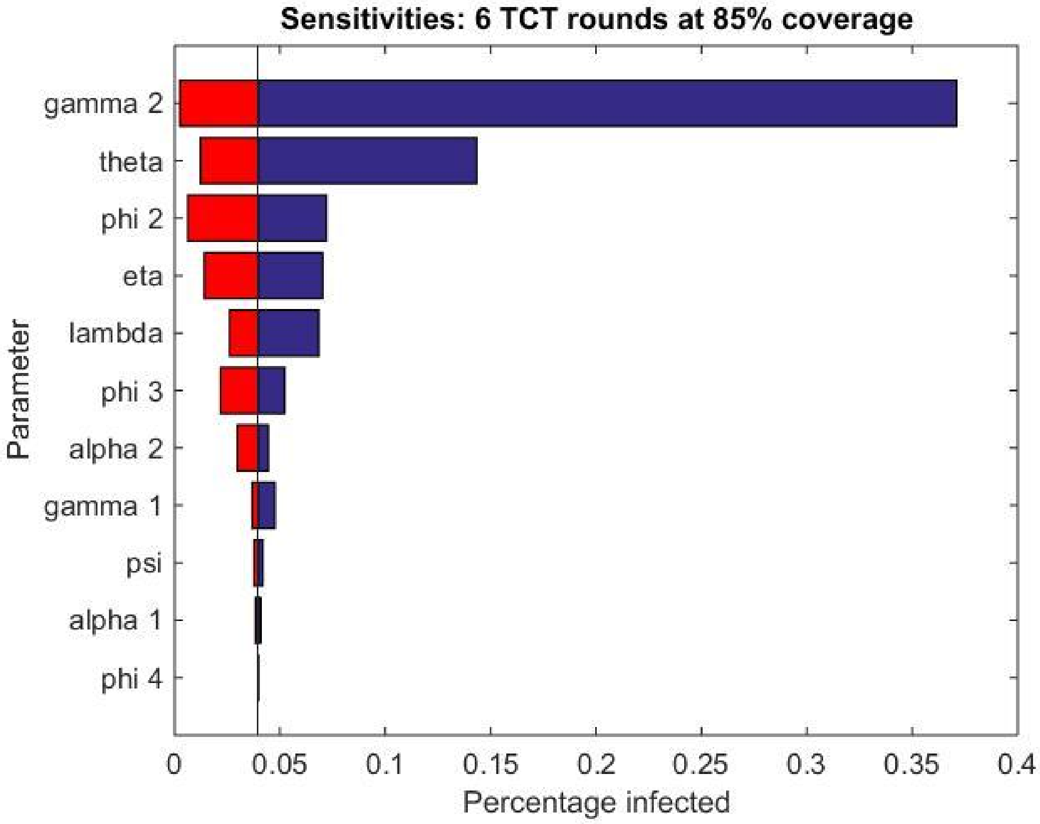
$$\frac{dR}{dt} = \gamma I$$



Sensitivity analysis



Sensitivity analysis



Lab 2

Take-home messages

- SIR models are one way to think about disease systems
- Type of model to use depends on specific question to be answered
- Be careful in how you communicate your model results
- Account for and be transparent about uncertainty